Modeling of rotational induction heating of nonmagnetic cylindrical billets

ARTICLE INFO

Keywords:
Induction heating
Magnetic field
Temperature field
Hard-coupled formulation
Higher-order finite element method

ABSTRACT

Induction heating of nonmagnetic cylindrical billets by rotation in uniform magnetic field produced by static permanent magnets is modeled. Numerical analysis of the process providing the distributions of magnetic and temperature fields is carried out in the monolithic formulation, using our own code based on higher-order finite element method. The methodology is illustrated by a typical example and the most important results are validated experimentally.

© 2011 Elsevier Inc. All rights reserved.

1. Introduction

Induction heating of metal billets is a widely spread industrial technology used mainly for their softening before hot forming. The conventional process is realized in classical harmonic current-carrying inductors where the billets are heated by the induced eddy currents. Due to its technical simplicity the process is very advantageous, but on the other hand, its efficiency is low and does not exceed 30–60%. This is because high thermal losses produced in inverters and inductors, which have to be carried off by appropriate cooling media.

In the last decade an alternative technology of heating was proposed consisting in rotation of such billets in time invariable magnetic field. Such a magnetic field can be produced either by a system of static DC currents-carrying coils [1,2] or static permanent magnets. The first case, however, is also characterized by production of losses, again in both inductors and inverter (the inverter being used for changing revolutions of the driving motor). That is why nowadays intensive attention is paid to the second case—heating of rotating billets in a field generated by permanent magnets. In such a case there are practically no coil or inductor losses and the only losses are mechanical, brought about by the drag torque generated by the interaction between the magnetic field in the system and eddy currents produced in the billet by rotation.

The paper deals with the numerical analysis of the operation parameters of this process. A typical arrangement is depicted in Fig. 1.

The aim of the paper is to find the following quantities in the form of functions of revolutions:

- average volumetric Joule losses in the heated cylinder,
- generalized coefficient of the convective heat transfer (by experiment),
- time evolution of the temperature of the heated charge, and
- the drag torque.
2. Continuous mathematical model

From the physical viewpoint, the task represents a doubly coupled problem. Its continuous mathematical model consists of two partial differential equations describing the distribution of magnetic and temperature fields in the system, whose coefficients are generally functions of the temperature. The first equation, describing the magnetic field in terms of magnetic vector potential \( A \), may be written in the form

\[
\text{curl}\left( \frac{1}{\mu} \text{curl} A - H_c \right) + \gamma \mathbf{v} \times \text{curl} A = 0,\]

where \( \gamma \) is the electrical conductivity, \( \mathbf{v} \) is the local velocity of rotation, and \( H_c \) is the coercive force of the permanent magnets (that is considered only in their domain, elsewhere it vanishes). A sufficiently distant artificial boundary is characterized by the Dirichlet condition \( A = 0 \).

The temperature field is described by the equation

\[
\text{div}(\lambda \cdot \text{grad} T) = \rho c \left( \frac{\partial T}{\partial t} + \mathbf{v} \cdot \text{grad} T \right) - p_i,\]

where \( \lambda \) is the thermal conductivity, \( \rho \) is the specific mass, and \( c \) denotes the specific heat. Finally, the symbol \( p_i \) denotes the time average internal sources of heat (the specific Joule losses) determined from the formula \( p_i = |\mathbf{J}_{\text{eddy}}|^2 / (2 \rho) \), where \( \mathbf{J}_{\text{eddy}} \) is the eddy current density following from the relation \( \mathbf{J}_{\text{eddy}} = \gamma \mathbf{v} \times A \). The boundary condition around the system respects both the convection and radiation. All physical parameters of the billet are generally functions of the temperature.

3. Numerical solution

The numerical solution of the problem is realized by a fully adaptive higher-order finite element method whose algorithms are implemented into codes Hermes2D [3] and Agros2D [4]. Both codes have been developed in our group for almost ten years.

The codes written in C++ are intended for monolithic numerical solution of systems of generally nonlinear and nonstationary second-order partial differential equations whose principal purpose is hard-coupled modeling of complex physical problems. While Hermes is a library containing the most advanced procedures and algorithms for the numerical processing of the task solved, Agros2D represents a powerful preprocessor and postprocessor. Both codes are freely distributable under the GNU General Public License. In the next paragraphs we will describe the principal algorithms that were used for solving the task and the most important features of the codes (some of them being quite original).

First, Eqs. (1) and (2) are reformulated in the sense of the weak solution. In Cartesian coordinates they have the forms

\[
\int_\Omega \frac{1}{\mu} \left( \frac{\partial A_z}{\partial x} + \frac{\partial A_x}{\partial y} \right) dS + \int_\Omega \gamma \left( A_x v_x \frac{\partial W}{\partial x} + A_y v_y \frac{\partial W}{\partial y} \right) dS = -\int_\Omega \frac{1}{\mu} \left( B_{xy} \frac{\partial W}{\partial x} - B_{yx} \frac{\partial W}{\partial y} \right) dS - \int_K \text{Kwdl},\]

\[
\int_\Omega \lambda \left( \frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} \right) dS + \int_\Omega \rho c_p \frac{\partial T}{\partial t} dS - \int_\Omega \rho c_p \left( v_x \frac{\partial W}{\partial x} + v_y \frac{\partial W}{\partial y} \right) dS + \int_\Gamma \alpha T dS = \int_\Gamma p_i dS + \int_\Gamma \left( \alpha T_0 + g \right) dS.\]
Here $\Omega$ denotes the cross section of the investigated area and $\Gamma$ is its boundary. Symbol $A_z$ is the $z$-component of magnetic vector potential, $v_x$ and $v_y$ are the components of velocity $\mathbf{v}$ in the corresponding directions, and finally $B_{rz}$ and $B_{ry}$ stand for the components of remanence $\mathbf{B}$. Symbol $x$ denotes the generalized coefficient of convective heat transfer, $T_0$ is temperature of external air, and $K$ and $g$ are appropriate constants. Finally, $w$ is a suitable testing function of polynomial character satisfying the boundary conditions of the problem.

The numerical solution of this model is then carried out by a fully adaptive higher-order finite element method ($hp$-FEM). It represents a modern version of the finite element method, which allows combining finite elements of variable size ($h$) and polynomial degree ($p$) in order to obtain fast exponential convergence of the solution. The principal features of our version of this method may be summarized into the following points.

- Solution of the system of PDEs is carried out monolithically, which means that the resultant numerical scheme is characterized by just one stiffness matrix.
- Fully automatic $hp$-adaptivity.
- Each physical field can be solved on quite a different mesh that best corresponds to its particulars. This is of great importance, for instance, for respecting skin effect in the electromagnetic field, boundary layers in the field of flow problems, etc. Special powerful higher-order techniques of mapping are then used to avoid any numerical errors in the process of assembly of the stiffness matrix.
- In nonstationary processes every mesh can change in time, in accordance with the real evolution of the corresponding physical quantities.
- No problems with the hanging nodes appearing on the boundaries of subdomains whose elements have to be refined [5]. Usually the hanging nodes bring about a considerable increase of the number of the degrees of freedom (DOFs). The code contains higher-order algorithms for respecting these nodes without any need of an additional refinement of the external parts neighboring with the refined subdomain.
- Curved elements able to replace curvilinear parts of any boundary by a system of circular or elliptic arcs. These elements mostly allow reaching highly accurate results near the curvilinear boundaries with very low numbers of DOFs.

Now we describe in more details the adaptivity techniques used for the processing of the problem. The automatic adaptation of $hp$-meshes significantly differs from the adaptivity in the standard FEM, since there exists a large number of refinement options for higher-order elements [6]. In $hp$-adaptivity, one can either increase the polynomial degree of an element without subdividing it in space or one can split an element in space and distribute the polynomial degrees in the subelements in multiple ways. This implies that traditional error estimates (one number per element) do not provide enough information to guide $hp$-adaptivity. One needs a better knowledge of the shape of the error function $e_{hp} = u - u_{hp}$. In principle, it might be possible to obtain this information from the estimates of higher derivatives, but this approach is not very practical.

In Hermes2D, the process of adaptivity starts to be applied at the moment when a local error of solution is higher than the acceptable tolerance. We will illustrate it by a general example: consider an equation

$$Lf = 0,$$

where $L$ denotes a differential operator and $f$ is a function whose distribution over some domain $\Omega$ is to be found. If $f$ is its approximation obtained by the numerical solution of (5), the absolute and percentage relative errors $\delta$ and $\eta$ are defined by the relations

$$\delta = f - f', \quad \eta = 100 \frac{\delta}{|f|}.$$

Other quantities that can be checked in this way are norms. Hermes2D works with the basic energetic norm given by the expression

$$\|e\| = \left| \int_{\Omega} \delta(L\delta) d\Omega \right|^{1/2}.$$

$L^2$ norm defined by the relation

$$\|e\|_{L^2} = \left| \int_{\Omega} \delta^2 d\Omega \right|^{1/2}$$

and $H^1$ norm given by the expression

$$\|e\|_{H^1} = \left| \int_{\Omega} \left( \delta^2 + (\text{grad} \delta) \cdot (\text{grad} \delta) \right) d\Omega \right|^{1/2}.$$

Unfortunately, the exact solution $f$ is known only in very simple analytically solvable cases. Moreover, there exists no general method that would provide a good estimation of the error for an arbitrary partial differential equation. That is why we work with the reference solution $f_{ref}$ instead, that is obtained either by a refinement of the whole mesh ($h$-adaptivity), by
enlargement of the polynomial degree (p-adaptivity) or by both above techniques (hp-adaptivity). In this manner we get the candidates for adaptivity even without knowledge of the exact solution $f$.

Fig. 2. Geometry of the investigated system (dimensions in mm).

Fig. 3. Arrangement of the experimental stand.

<table>
<thead>
<tr>
<th>Item</th>
<th>Material</th>
<th>Characteristic values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnetic circuit</td>
<td>Carbon steel 12 040</td>
<td>$\mu_s$, see Fig. 4</td>
</tr>
<tr>
<td>Permanent magnets</td>
<td>NeFeB magnets VMM 10</td>
<td>$B_r = 1.4$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$T_w = 150 , ^\circ C$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\mu_f = 1.21$</td>
</tr>
<tr>
<td>Heated pipe</td>
<td>Aluminum</td>
<td>$\mu_r = 1$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$T_0 = 20 , ^\circ C$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\gamma$, see Fig. 5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\nu$, see Fig. 6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\rho c$, see Fig. 7</td>
</tr>
</tbody>
</table>
Fig. 4. Magnetisation characteristic $B=f(H)$ of carbon steel 12040.

Fig. 5. Dependence $\gamma = \gamma(T)$ of pure aluminum [4].

Fig. 6. Dependence $\lambda = \lambda(T)$ of pure aluminum [4].
In our case, we used the $H^1$ norm. Before the adaptivity loop is applied, the code must initialize the refinement selector that determines how the elements will be refined. The selector performs the following steps:

- selection of the candidates for refinement,
- computation of their local errors, which is realized by projecting the reference solution on their FE spaces,
- computation of the number of the degrees of freedom for every candidate,
- evaluation of the score for each candidate and sorting the candidates according to their values,
- selection of the candidate with the highest score.

As mentioned before, the adaptivity algorithm in Hermes needs an actual mesh solution and another solution realized on globally refined mesh (the reference solution). These solutions are subtracted in each adaptivity step in order to obtain an error estimate (as a function). This function is used to decide which elements need to be refined and in which way. Hence the adaptivity loop begins with refining the mesh globally and calculating the reference solution.

![Graph](image1)

**Fig. 7.** Dependence $\rho c = \rho c(T)$ of pure aluminum [4].

![Image](image2)

**Fig. 8.** Initial rough (white lines) and final (grey lines) meshes for calculation of magnetic field in the system.
If the error is higher than a given threshold the adaptation process is started. The calculated local error in the candidate is first weighted with respect to the way of adaptivity that should be used. This weight \( w \) is:

- \( w = 2 \) for the \( h \)-adaptivity,
- \( w = 1 \) for the \( p \)-adaptivity,
- \( w = \sqrt{2} \) for the \( hp \)-adaptivity.

The score of a candidate is given by the formula

\[
s = \log_{10} \frac{d_0}{d} \left( \frac{d}{d_0} \right)^n
\]  

\( (10) \)

---

**Fig. 9.** Distribution of the magnetic field in the system for \( n = 3000 \) rev./min. after 300 s of heating.

**Fig. 10.** Average specific Joule losses \( p_j \) as a function of revolutions \( n \).
where $e$ is the estimated error in the candidate and $d$ denotes its number of DOFs, $e_0$ and $d_0$ are given parameters and $\xi$ is a convergence exponent.

4. Illustrative example

The described technology was applied to a hollow aluminum pipe rotating in magnetic field produced by permanent magnets. The cross section of the system is depicted in Fig. 2. Its length in the direction of the longitudinal axis is 40 mm. The complete arrangement of the stand used for experimental verification of the results is shown in Fig. 3.

Fig. 11. Experimentally determined dependence $a = a(n)$.

Fig. 12. Distribution of the temperature in the pipe for $n = 3000$ rpm after 300 s of heating.
The basic physical properties of the main parts of the system in Fig. 2 are listed in Table 1. Other necessary material characteristics are in Figs. 4–7 [7].

First, we proposed a rough mesh for the computation of magnetic field, the highest degree of polynomials approximating the distribution of magnetic vector potential in particular cells being chosen seven. After several adaptive steps we obtained the final mesh satisfying the prescribed tolerance. Both meshes are depicted in Fig. 8.

The above distribution was used for determination of eddy currents in the hollow pipe, corresponding Joule losses, temperature rise, and also for finding the most important mechanical characteristics.

Distribution of the magnetic field in the system for \( n = 3000 \) rev./min. after 300 s of heating is depicted in Fig. 9.

Fig. 15 shows the dependence of the calculated average volumetric Joule losses in the heated aluminum pipe on the number of the revolutions per minute. The dependence is almost parabolic, corresponding to a linear system. This is because the influence of the nonlinear magnetic circuit is not too significant.

For the thermal computations we experimentally found the dependence of the generalised coefficient \( a = a(n) \) of convection as a function of revolutions per minute. The coefficient was determined indirectly from the measurements of the surface temperature of the heated pipe using a precise Fluke thermocamera. The measured dependence is in Fig. 11. These coefficients were used for computation of the time evolution of the average temperature of the pipe for different revolutions.

![Graph showing temperature evolution over time for \( n = 6000 \) rev./min. with \( a = 88 \text{ W/(m}^2\text{K}) \).](image1)

**Fig. 13.** Time evolution of the average temperature of the pipe for \( n = 6000 \) rev./min. (\( a = 88 \text{ W/(m}^2\text{K}) \)).

![Graph showing calculated dependence of drag torque \( T_d \) on rpm.](image2)

**Fig. 14.** Calculated dependence of the drag torque \( T_d \) on rpm.
One example (distribution of the temperature in the pipe for $n = 3000$ rpm after 300 s of heating) is depicted in Fig. 12, another example, showing the time evolution of the average temperature of the pipe for $n = 6000$ rpm is in Fig. 13.

Of great importance is also the dependence of the drag torque $T_d$ acting on the rotating pipe. This torque can easily be calculated from the distribution of magnetic field and eddy currents produced in the pipe. Its dependence on the revolutions is depicted in Fig. 14.

The graph is relevant only in the interval $(0, 23700)$ rpm, because the right value represents the maximum revolutions of the driving motor (the values in the rectangle being, therefore, unrealistic).

Finally, the last two graphs in Figs. 10 and 16 show the power consumption and heating efficiency of the whole process.

5. Conclusion

The results show that induction heating of long billets realised by their rotation in a time invariable magnetic field represents a promising technology characterised by several advantages in comparison with a system working with direct
current carrying coils. First, no extra source of electric energy for production of magnetic field is needed. Second, in order to obtain the same average Joule losses in the heated object (and, consequently, the same velocity of heating) extreme current densities of the direct current would be necessary. This would require a corresponding source of energy and additional intensive cooling of the field coils.

Next work in the field will be aimed at the analysis of the last mentioned system and also at the inclusion of the front effects due to small axial length of the system.

Acknowledgments

The financial support of the Grant Agency of the Czech Republic (project No. P102/10/0216), the European Regional Development Fund and Ministry of Education, Youth and Sports of the Czech Republic under the project No. CZ.1.05/2.1.00/03.0094: Regional Innovation Centre for Electrical Engineering (RICE) and Research Plan MSM 6840770017 is gratefully acknowledged.

References